Lecture 14 - March 4

General Trees, Binary Trees

Initializing a Generic Array
Recursive Definitions of (Binary) Trees
Trees in Java: Construction, Depth

Announcements/Reminders

- ProgTest1 results to be released by Friday, Mar 14
- Makeup Lecture (on ADTs, Stacks) posted
- Assignment 3 (on linked Trees) released
- WrittenTest guide and example questions to be release
- Lecture notes template, Office Hours, TA Contact

General Trees: Recursive Definition

- root
- size

Case 0: Empty Tree

ϕ

root → null

size == 0

Case 1: A singleton tree

root →

children → null

implementation decision

ArrayList<TreeNode>

empty

(.size() == 0)

Case 2: ≥ 2 nodes

root →

c.

0 1 2 MAX-1

. . .

TreeNode[]

c.

c.

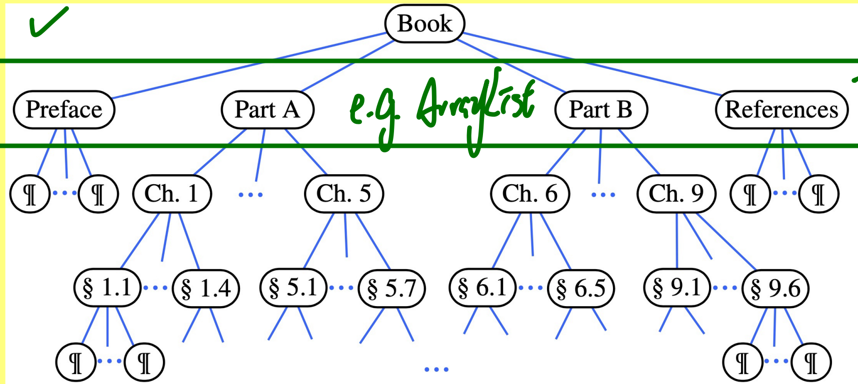
...

...

subtree rooted at 1st child

→ subtree rooted at 2nd child

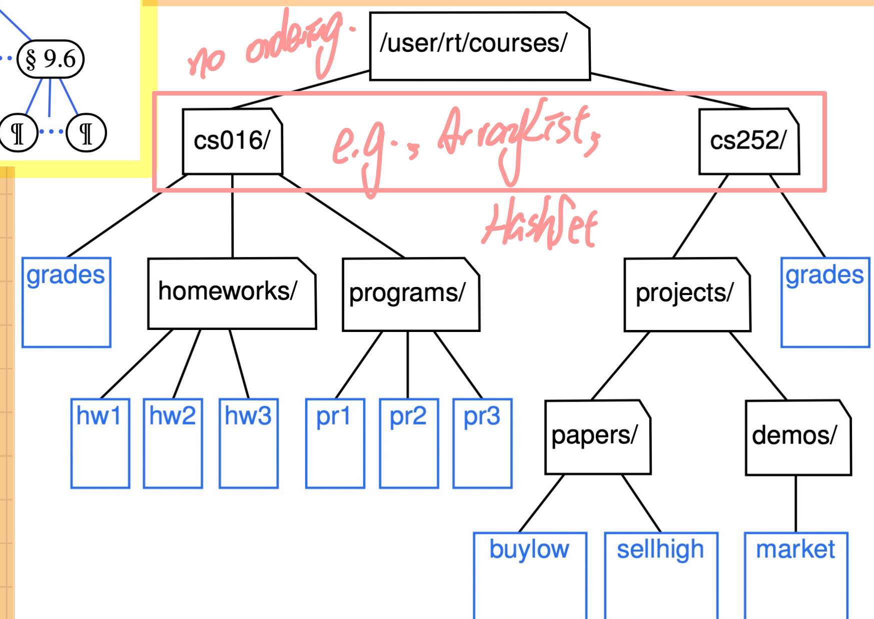
General Trees: Ordered vs. Unordered Trees



e.g. ArrayList

→ there's a linear, logical order between a nodes child nodes

no ordering.



e.g., ArrayList, HashSet

red

↳ tree

$n.noc \le n.children.length$

$n.children.length == 10$

$n.noc == \cancel{0}\ \cancel{1}\ 2$

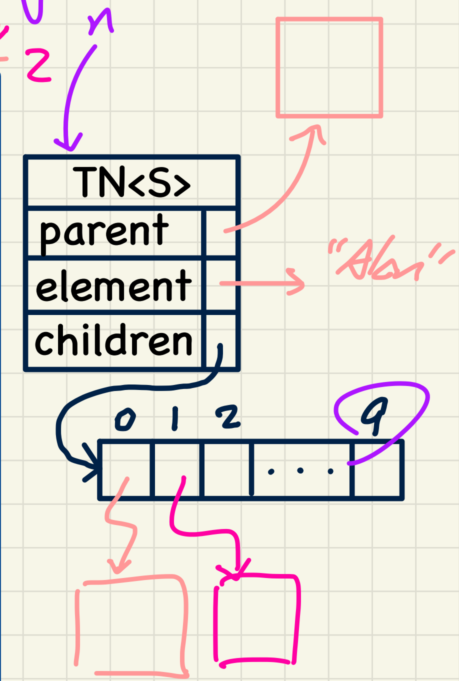
Generic, General Tree Nodes

```
public class TreeNode<E> {
  private E element; /* data object */
  private TreeNode<E> parent; /* unique parent node */
  private TreeNode<E>[] children; /* list of child nodes */

  private final int MAX_NUM_CHILDREN = 10; /* fixed max */
  private int noc; /* number of child nodes */

  public TreeNode(E element) {
    this.element = element;
    this.parent = null;
    this.children = (TreeNode<E>[])
      Array.newInstance(this.getClass(), MAX_NUM_CHILDREN);
    this.noc = 0;
  }
  public E getElement() { ... }
  public TreeNode<E> getParent() { ... }
  public TreeNode<E>[] getChildren() { ... }

  public void setElement(E element) { ... }
  public void setParent(TreeNode<E> parent) { ... }
  public void addChild(TreeNode<E> child) { ... }
  public void removeChildAt(int i) { ... }
}
```



Compare:

+ prev ref.

+ next ref.

in a DLN.

Instantiating Generic Structures in Java

```
class ArrayStack<E> {
  private E[] data;
  ...
  public ArrayStack<E>() {

  }
}
```

S

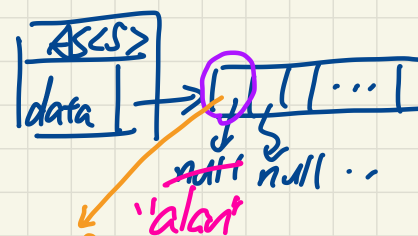
data = new E[10]; ✗

data = (E[]) new Object[10]; ✓

AS<String> s = new AS<>();

s.push(23); ✗

s.push("alan"); s



ST: String

DT: String

```
class TreeNode<E> {
  private TreeNode<E>[] children;
  ...
  public TreeNode<E>() {

  }
}
```

E is wrapped within the TreeNode class

children = (TN<E>[]) new object[10]; ✗

RT: Object[]

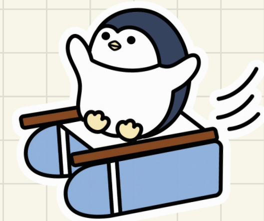
children = (TN<E>[]) Array.newInstance(this.getClass(), 10);

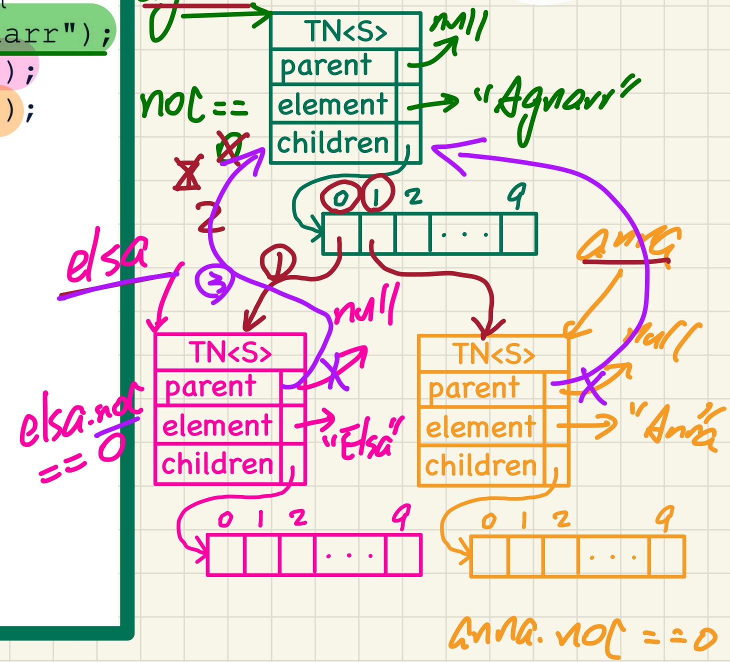
Tracing: Constructing a Tree

```java
@Test
public void test_general_trees_construction() {
  TreeNode<String> agnarr = new TreeNode<>("Agnarr");
  TreeNode<String> elsa = new TreeNode<>("Elsa");
  TreeNode<String> anna = new TreeNode<>("Anna");

agnarr.addChild(elsa);
agnarr.addChild(anna);
elsa.setParent(agnarr);
anna.setParent(agnarr);

  assertNull(agnarr.getParent());
  assertTrue(agnarr == elsa.getParent());
  assertTrue(agnarr == anna.getParent());
  assertTrue(agnarr.getChildren().length == 2);
  assertTrue(agnarr.getChildren()[0] == elsa);
  assertTrue(agnarr.getChildren()[1] == anna);
}
```





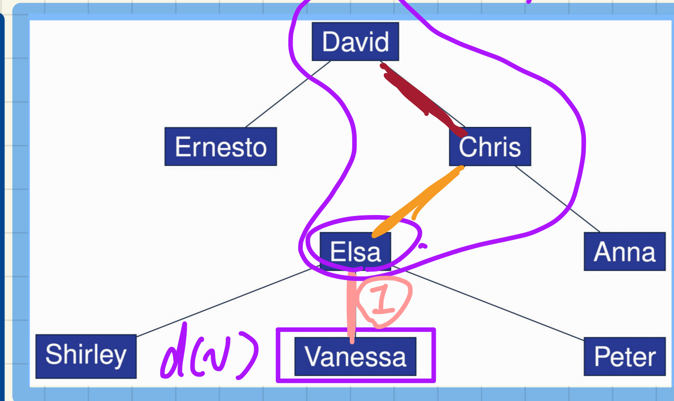
Tracing: Computing a Node's Depth

root of some subtree

```java
public int depth(TreeNode<E> n) {
  if(n.getParent() == null) {
    return 0;
  }
  else {
    return 1 + depth(n.getParent());
  }
}
```

depth of root is 0.

d(v's parent)




```java
@Test
public void test_general_trees_depths() {
  ... /* constructing a tree as shown above */
  TreeUtilities<String> u = new TreeUtilities<>();
  assertEquals(0, u.depth(david));
  assertEquals(1, u.depth(ernesto));
  assertEquals(1, u.depth(chris));
  assertEquals(2, u.depth(elsa));
  assertEquals(2, u.depth(anna));
  assertEquals(3, u.depth(shirley));
  assertEquals(3, u.depth(vanessa));
  assertEquals(3, u.depth(peter));
}
```

depth(vanessa)

↳ .getP != null

= 1 + depth(Elsa)

↳ .getP != null

= 1 + 1 + depth(Chris)

= 1 + 1 + 1 + $\boxed{\text{depth(David)}}$

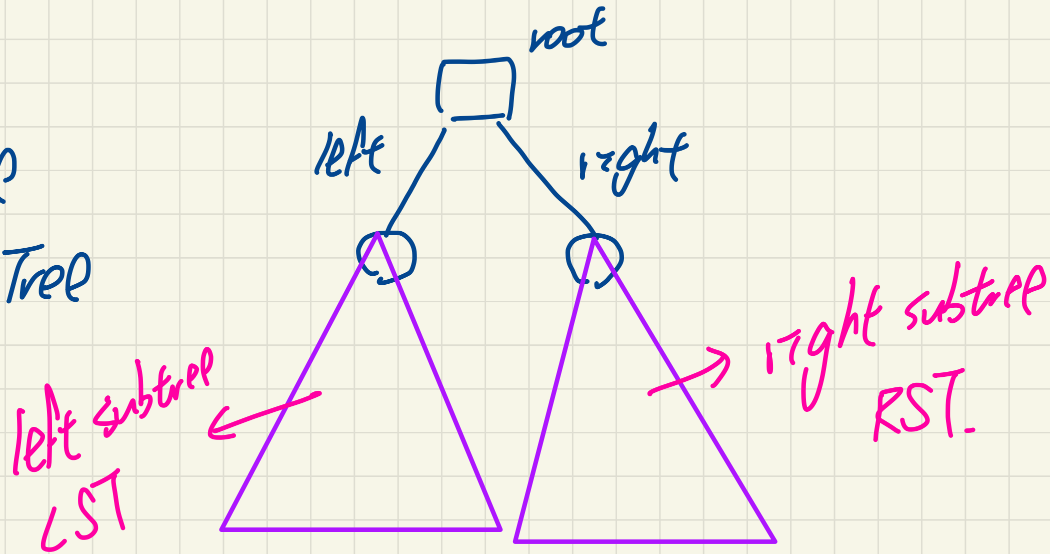
0

= 3

Binary Trees: Recursive Definition

- root
- size

Case 2: ≥ 2 nodes



Case 0: Empty Tree

Case 1: Singleton Tree

Deriving the Sum of a Geometric Sequence

Initial Term: I

Common Factor: r

Number of Terms: k

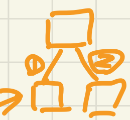
$$\overset{2^0}{1} + \underset{*2}{2} + \underset{*2}{4} + 8 + 16 + \cdots + \overset{2^{10}}{1024}$$

$k = 11$

$$S_k = \overset{I \cdot r^0}{I} + \boxed{I \cdot r + I \cdot r^2 + I \cdot r^3 + \cdots + I \cdot r^{k-1}}$$

$$r \cdot S_k = \boxed{I \cdot r + I \cdot r^2 + I \cdot r^3 + \cdots + I \cdot r^{k-1}} + I \cdot r^k$$

$$r \cdot S_k - S_k = (r-1) \cdot S_k = I \cdot r^k - I = I \cdot (r^k - 1)$$



$$S_k = \frac{I \cdot (r^k - 1)}{r - 1}$$

For BT: $r = 2$, $I = 1$ root

$$S_k = \frac{1 \cdot (2^k - 1)}{2 - 1} = 2^k - 1$$

depth of "bottom" node